Energy Efficiency Optimization For Two-way Relay Channels

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Introduction

Due to the global warming and the operators' increasing operational cost, energy efficiency (EE) has drawn increasing attention and been viewed as a new optimization criterion for green wireless communication systems.

On the other hand, the wireless two-way relay channel (TWRC) is proposed to improve the system spectral efficiency (SE) as well as EE.

EE optimization has been widely discussed in one-way transmissions, the basic ideas are mainly based on the convex-concave fractional programs. There are limited works considering the EE of TWRC, so this paper proposes an algorithm based on the nested optimization to solve it. Compared with optimizing by fractional programming directly, our algorithm has more insights on the relationship of the transmit power of different nodes and the constraints.

System Model

Consider a TWRC consisting of two source nodes A and B exchanging information with the assist of a relay node R.

System capacity: $C(P_A, P_B, P_R) = \frac{1}{2}\left[\min\left(C_A(P_A), C_B(P_R)\right) + \min\left(C_B(P_B), C_A(P_R)\right)\right]$

where $C_i(P) = W\log_2(1 + P\gamma_i),\ i \in \{A, B\}$, W is the bandwith, and γ_i is the channel gain to noise ratios of the two channel from i to R.

Power consumption model: $\frac{P_i}{\eta} + P_{c,i}$, $i \in \{A, B, R\}$. η is the power conversion efficiency, $P_{c,i}$ is the circuit power.

System EE: $EE(P_A, P_B, P_R) = \dfrac{C(P_A, P_B, P_R)}{\frac{1}{2\eta}(P_A + P_B + P_R) + P_c}$,

where $P_c = \frac{1}{2}\sum_{i=A,B,R} P_{c,i}$ represents the system total circuit power.

As our objective is to maximize the EE, considering with node i's power constraint $P_{i,\max}$, the optimization problem can be expressed as

$$\max_{P_A, P_B, P_R} EE(P_A, P_B, P_R)$$
$$s.t.\quad 0 \le P_A \le P_{A,\max},\quad 0 \le P_B \le P_{B,\max},\quad 0 \le P_R \le P_{R,\max}$$

In the following, we will solve the unconstrained EE optimization at first through nested optimization, then obtain the constrained solutions based on it.

Unconstrained Optimization

Firstly, consider the the unconstrained optimization problem $\max_{P_A,P_B,P_R} EE(P_A, P_B, P_R)$. Employ the nested optimization, rewriting it as

$$\max_{P_{total}} \frac{\max_{P_A+P_B+P_R=P_{total}} C(P_A, P_B, P_R)}{\frac{1}{2\eta}P_{total} + P_c}$$

Define $C_0(P_{total}) = \max_{P_A+P_B+P_R=P_{total}} C(P_A, P_B, P_R)$, $EE_0(P_{total}) = C_0(P_{total}) \Big/ \left(\frac{1}{2\eta}P_{total} + P_c\right)$.

Then we can express the unconstrained ptimization problem as $\max_{P_{total}} EE_0(P_{total})$. We firstly try to get the close-form expressions of $C_0(P_{total})$.

Lemma 1: With fixed P_{total}, to maximize system capacity, the optimal solution must satisfy one of the following three situations:

$P_A/P_R = \gamma_B/\gamma_A$ and $P_B/P_R < \gamma_A/\gamma_B$; (1)

$P_A/P_R < \gamma_B/\gamma_A$ and $P_B/P_R = \gamma_A/\gamma_B$; (2)

$P_A/P_R = \gamma_B/\gamma_A$ and $P_B/P_R = \gamma_A/\gamma_B$. (3)

Lemma 2: With fixed P_{total} , the optimal power allocation is:

1. When $\gamma_B/\gamma_A < \left(\sqrt{5}-1\right)/2$ and $P_{total} > \dfrac{\gamma_A^2 + \gamma_A\gamma_B + \gamma_B^2}{\gamma_A\left(\gamma_A^2 - \gamma_A\gamma_B - \gamma_B^2\right)} \overset{\Delta}{=} P_1$

$$\begin{cases} P_A = \dfrac{\gamma_B}{2(\gamma_A+\gamma_B)}P_{total} - \dfrac{\gamma_B}{2\gamma_A(\gamma_A+\gamma_B)} \\ P_R = \dfrac{\gamma_A}{2(\gamma_A+\gamma_B)}P_{total} - \dfrac{1}{2(\gamma_A+\gamma_B)} \\ P_B = \dfrac{1}{2}P_{total} + \dfrac{1}{2\gamma_A} \end{cases}$$

2. When $\gamma_B/\gamma_A > \left(\sqrt{5}+1\right)/2$ and $P_{total} > \dfrac{\gamma_A^2 + \gamma_A\gamma_B + \gamma_B^2}{\gamma_B\left(\gamma_B^2 - \gamma_A\gamma_B - \gamma_A^2\right)} \overset{\Delta}{=} P_2$

$$\begin{cases} P_A = \dfrac{1}{2}P_{total} + \dfrac{1}{2\gamma_B} \\ P_R = \dfrac{\gamma_B}{2(\gamma_A+\gamma_B)}P_{total} - \dfrac{1}{2(\gamma_A+\gamma_B)} \\ P_B = \dfrac{\gamma_A}{2(\gamma_A+\gamma_B)}P_{total} - \dfrac{\gamma_A}{2\gamma_B(\gamma_A+\gamma_B)} \end{cases}$$

3. In other situation,

$$\begin{cases} P_A = \dfrac{\gamma_B^2}{\gamma_A^2+\gamma_A\gamma_B+\gamma_B^2}P_{total} \\ P_R = \dfrac{\gamma_A\gamma_B}{\gamma_A^2+\gamma_A\gamma_B+\gamma_B^2}P_{total} \\ P_B = \dfrac{\gamma_A^2}{\gamma_A^2+\gamma_A\gamma_B+\gamma_B^2}P_{total} \end{cases}$$

Theorem 1: $C_0(P_{total})$ can be expressed as follows upon different γ_B/γ_A :

1. When $\gamma_B/\gamma_A < \left(\sqrt{5}-1\right)/2$, $C_0(P_{total}) = \begin{cases} C_c(P_{total}), P_{total} \le P_1 \\ C_a(P_{total}), P_{total} > P_1 \end{cases}$

2. When $\gamma_B/\gamma_A > \left(\sqrt{5}+1\right)/2$, $C_0(P_{total}) = \begin{cases} C_c(P_{total}), P_{total} \le P_2 \\ C_b(P_{total}), P_{total} > P_2 \end{cases}$

3. When $\left(\sqrt{5}-1\right)/2 \le \gamma_B/\gamma_A \le \left(\sqrt{5}+1\right)/2$, $C_0(P_{total}) = C_c(P_{total})$

where

$$C_a(P_{total}) = \frac{1}{2}W\left[\log_2\left(\frac{2\gamma_A+\gamma_B}{2(\gamma_A+\gamma_B)} + \frac{\gamma_A\gamma_B}{2(\gamma_A+\gamma_B)}P_{total}\right) + \log_2\left(\frac{2\gamma_A+\gamma_B}{2\gamma_A} + \frac{1}{2}\gamma_B P_{total}\right)\right]$$

$$C_b(P_{total}) = \frac{1}{2}W\left[\log_2\left(\frac{\gamma_A+2\gamma_B}{2(\gamma_A+\gamma_B)} + \frac{\gamma_A\gamma_B}{2(\gamma_A+\gamma_B)}P_{total}\right) + \log_2\left(\frac{\gamma_A+2\gamma_B}{2\gamma_B} + \frac{1}{2}\gamma_A P_{total}\right)\right]$$

$$C_c(P_{total}) = \frac{1}{2}W\left[\log_2\left(1 + \frac{\gamma_A\gamma_B^2}{\gamma_A^2+\gamma_A\gamma_B+\gamma_B^2}P_{total}\right) + \log_2\left(1 + \frac{\gamma_A^2\gamma_B}{\gamma_A^2+\gamma_A\gamma_B+\gamma_B^2}P_{total}\right)\right]$$

Theorem 2: $EE_0(P_{total})$ is a strictly quasi-concave function.

So $EE_0(P_{total})$ is first strictly increasing and then strictly decreasing as a function of P_{total}, the one and the only one optimal solution P_{total}^* can be obtained efficiently by bisection. Then the optimal power allocation P_A^* , P_B^* , P_R^* can be calculated according to Lemma 2.

Constrained optimization

After solving the unconstrained optimal solution P_A^* , P_B^*, P_R^* and the corresponding P_{total}^* , we move on to solve the original constrained problem.

If P_A^*, P_B^*, P_R^* all fulfill the constraints, they are the optimal constrained solutions. Otherwise, we will obtain the optimal solutions from the quasi-concavity of EE by the following steps, the basic idea is to confirm several nodes' optimal power and reduce the range of the other nodes' power optimization interval.

Lemma 3: The optimal solution of EE optimization problem for TWRC must satisfy one of the situations in (1)(2)(3).

Then from (1)(2)(3) the system capacity and EE can be rewritten as $C(P_A, P_B) = \frac{1}{2}\left[C_A(P_A) + C_B(P_B)\right]$,

$EE(P_A, P_B) = \dfrac{C(P_A, P_B)}{\frac{1}{2\eta}\left(P_A + P_B + P_R(P_A, P_B)\right) + P_c}$,where $P_R(P_A, P_B) = \max\left(P_A\gamma_A/\gamma_B, P_B\gamma_B/\gamma_A\right)$.

Step 1: According to the power constraints $P_{i,\max}$, correspondingly calculate three total transmit power $P_{total,i}$ based on Lemma 2. Pick out the node $k = \arg\min_i\left(P_{total,i}\right)$, and calculate P_A^0 , P_B^0, P_R^0 from $P_{total,k}$. From the quasi-concavity of EE, $P_{k,\max}$ is is the constrained optimal power for node k.

Step 2:

1)k is a source node, consider the case k=A.

a)If $P_B^0 < P_R^0\gamma_A/\gamma_B$, optimize

$$\max_{P_B} \frac{\frac{1}{2}W\left[C_A(P_{A,\max}) + C_B(P_B)\right]}{\frac{1}{2\eta}\left(P_{A,\max} + P_B + P_R^0\right) + P_c}$$
$$s.t.\ P_B^0 \le P_B \le \min\left(P_R^0\gamma_A/\gamma_B, P_{B,\max}\right)$$

If the solution is $P_R^0\gamma_A/\gamma_B$, refresh $P_B^0 = P_R^0\gamma_A/\gamma_B$ and turn to b). Otherwise, the derived P_B and P_R^0 are constrained optimal.

b)If $P_B^0 = P_R^0\gamma_A/\gamma_B$, optimize

$$\max_{P_B} \frac{\frac{1}{2}W\left[C_A(P_{A,\max}) + C_B(P_B)\right]}{\frac{1}{2\eta}\left(P_{A,\max} + P_B + P_B\gamma_B/\gamma_A\right) + P_c}$$
$$s.t.\ P_B^0 \le P_B \le \min\left(P_{R,\max}\gamma_A/\gamma_B, P_{B,\max}\right)$$

the derived P_B and corresponding $P_R = P_B\gamma_B/\gamma_A$ are constrained optimal.

2)If k =R.

a)If P_A^0, P_B^0 , $P_{R,\max}$ fulfill (3), they are the optimal constrained solutions.

b)If (1)or(2) is fulfilled, consider the case (1). P_A^0 is optimal, and we need optimize P_B from

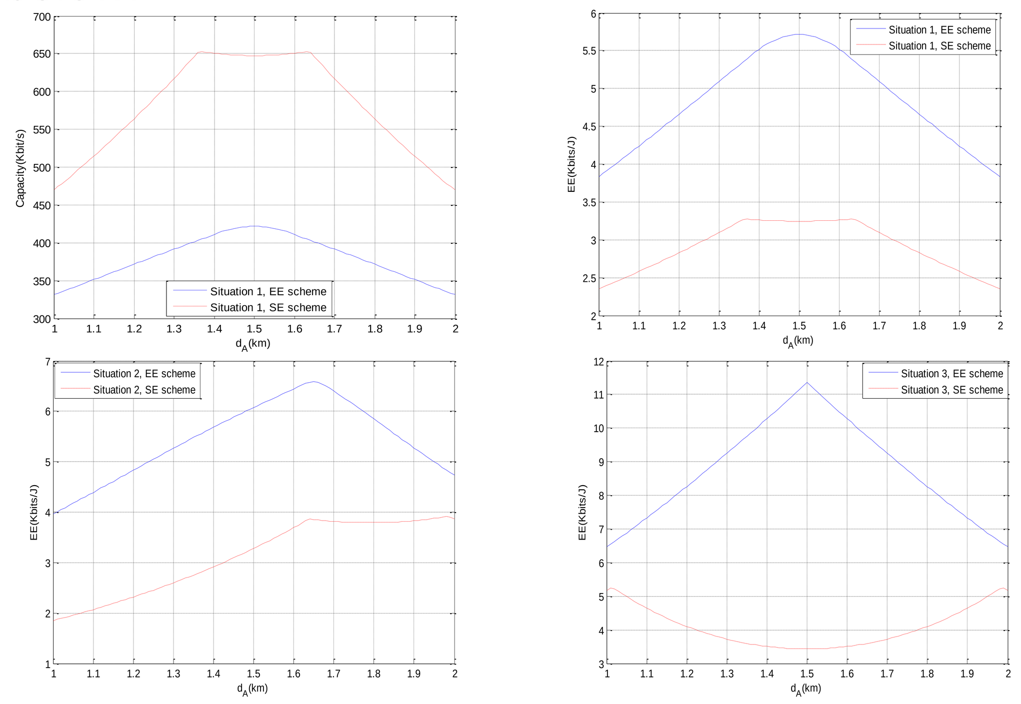
$$\max_{P_B} \frac{\frac{1}{2}W\left[C_A(P_A^0) + C_B(P_B)\right]}{\frac{1}{2\eta}\left(P_A^0 + P_B + P_{R,\max}\right) + P_c}$$
$$s.t.\ P_B^0 \le P_B \le \min\left(P_{R,\max}\gamma_A/\gamma_B, P_{B,\max}\right)$$

Simulation Results

Set pathloss as $128.1 + 37.6\log_{10} d_i dB$, noise power is -100dBm, W=200KHz,$\eta = 0.38$. And we set three situations as follows.

	Node A	Node R	Node B
Situation 1	Base station	Relay	Base station
Situation 2	Base station	Relay	Mobile Phone
Situation 3	Mobile phone	Relay	Mobile Phone

The capacity comparison for situation 1 and EE comparison for each comparison are shown as below:



We can conclude that:

1. Our scheme is efficiency on EE, but has a capacity loss. We should find a trade-off in practice.
2. The relay should be located closer to the node with stricter power constraint.
3. Reducing the circuit power is also a very important part for the EE goal.

And for these simulations above, our proposed scheme need 0.2532s on average and the convex-concave fractional programs need 0.7952s, so the complexity of the algorithm proposed by this paper is much less than the convex-concave fractional programs.